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AN INTERFACIAL CRACK IN A TRANSVERSELY ISOTROPIC COMPOSITE MEDIUM†

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The three-dimensional stressed state in the neighbourhood of the tip of a plane semi-infinite crack located at the interface between two different kinds of transversely isotropic materials is investigated. Its exact analytic solution is constructed by reducing the problem to a Wiener–Hopf matrix equation, the singularity index of the elastic field is found and the stress intensity factors are determined.

A similar three-dimensional problem has previously been studied for the case of isotropic materials [1]. Problems on interfacial cracks in anisotropic media have only been investigated in a two-dimensional formulation (see, for example, [2, 3]).

1. Consider an elastic space consisting of two transversely-isotropic homogeneous half-spaces $z \geqslant 0$ and $z \leqslant 0$, henceforth denoted by the subscripts 1 and 2. Suppose a crack in the form of the half-plane $\{(x, y): x < 0, |y| < \infty\}$ is located at the interface of the media $z = 0$ and that a self-balanced normal load $p(x, y)$, which is symmetric about the x axis, is applied to the crack edges. There is ideal mechanical contact between the materials on the remaining part of the boundary.

Assuming that the isotropy planes of the materials of the half-spaces are parallel to the plane of the crack and that there are no bulk forces, the problem reduces to solving the equilibrium equations in the displacements [4] subject to the following conditions on the boundary $z = 0$

$$\tau_{xzj} = \tau_{yzj} = 0, \quad \sigma_{zj} = -p(x,y) \tag{1.1}$$

$$(j = 1,2; \quad x < 0, \quad |y| < \infty)$$

$$\tau_{xz1} = \tau_{xz2}, \quad \tau_{yz1} = \tau_{yz2}, \quad \sigma_{z1} = \sigma_{z2} \tag{1.2}$$

$$u_1 = u_2, \quad v_1 = v_2, \quad w_1 = w_2 \quad (x > 0, \ |y| < \infty) \tag{1.3}$$

where u_j, v_j, w_j are the displacements along the x, y and z axes and τ_{xzj}, τ_{yzi}, σ_{zj} are the components of the stress tensors.

In addition, it is necessary to take account of the further requirement that the stresses should decrease at infinity and the condition that the potential energy of deformation must be bounded in the neighbourhood of the crack tip.

Applying a two-dimensional Fourier transformation with respect to the variables x and y to the equilibrium equations we obtain the following representation of the displacements

$$\left\| \begin{matrix} u_j(x,y,z) \\ v_j(x,y,z) \\ w_j(x,y,z) \end{matrix} \right\| = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{L_\lambda} \left\| \begin{matrix} U_j(\lambda,\mu,z) \\ V_j(\lambda,\mu,z) \\ W_j(\lambda,\mu,z) \end{matrix} \right\| e^{-i(\lambda x + \mu y)} d\lambda d\mu \tag{1.4}$$

where

$$U_j(\lambda,\mu,z) = \sum_{k=1}^{3} A_{jk} e^{-\omega_{jk}\gamma|z|}, \quad V_j(\lambda,\mu,z) = \sum_{k=1}^{3} A_{jk} p_k e^{-\omega_{jk}\gamma|z|}$$
$$W_j(\lambda,\mu,z) = (-1)^j \frac{i\gamma}{\lambda} \sum_{k=1}^{2} A_{jk} \omega_{jk} q_{jk} e^{-\omega_{jk}\gamma|z|} \quad (j = 1,2) \tag{1.5}$$

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$$A_{jk} \equiv A_{jk}(\lambda,\mu),\quad p_1 = p_2 = \mu/\lambda,\quad p_3 = -\lambda/\mu,\quad \gamma = (\lambda^2+\mu^2)^{1/2}$$
$$q_{jk} = (C_{13}^{(j)} + C_{44}^{(j)})/(C_{33}^{(j)}\omega_{jk}^2 - C_{44}^{(j)})\quad (j,k=1,2)$$
$$\omega_{jk}^2 = \alpha_j \mp (\alpha_j^2 - \beta_j)^{1/2}\quad (k=1,2),\quad \omega_{j3} = (C_{66}^{(j)}/C_{44}^{(j)})^{1/2}$$
$$\alpha_j = (C_{11}^{(j)}C_{33}^{(j)} - C_{13}^{(j)2} - 2C_{13}^{(j)}C_{44}^{(j)})/(2C_{33}^{(j)}C_{44}^{(j)}),\quad \beta_j = C_{11}^{(j)}/C_{33}^{(j)}$$

According to [4], the elasticity constants, $C_{mn}^{(j)}$, of the half-spaces satisfy the inequalities

$$C_{11}^{(j)} > 0,\quad C_{11}^{(j)} > C_{12}^{(j)},\quad C_{44}^{(j)} > 0,\quad C_{33}^{(j)}(C_{11}^{(j)} + C_{12}^{(j)}) > 2C_{13}^{(j)^2} \tag{1.6}$$

Henceforth, the single-valued branch of the function $\gamma = (\lambda^2 + \mu^2)^{1/2}$ is considered which is defined in the complex plane with cuts along the rays $(-i\infty, -i|\mu|)$ and $(i|\mu|, i\infty)$ and the positive values are taken when λ is real. The integration contour L_λ in (1.4) is located in the strip $-|\mu| < \operatorname{Im}\lambda < 0$.

The characteristic numbers ω_{j1} and ω_{j2} for different pairs of materials may be assumed to be real or complex conjugate quantities. Those of them for which $\operatorname{Re}\omega_{jk} > 0$ are used in the sums in (1.5).

Conditions (1.1) and (1.2) enable one to express the constants A_{2k} in terms of A_{1k}. After this, using the mixed boundary conditions (1.1) for $j = 1$ and (1.3), we find that $A_{j3} = 0$ and the quantities A_{11} and A_{12} are determined from the following system of coupled integral equations

$$\begin{aligned}
&(2\pi)^{-1/2}\int_{L_\lambda}\gamma[\omega_{11}C_1(\lambda,\mu)+\omega_{12}C_2(\lambda,\mu)]e^{-i\lambda x}d\lambda = 0\quad (x<0)\\
&(2\pi)^{-1/2}\int_{L_\lambda}\frac{i\gamma^2}{\lambda}[C_1(\lambda,\mu)+C_2(\lambda,\mu)]e^{-i\lambda x}d\lambda = -p^*(x,\mu)\quad (x<0)\\
&(2\pi)^{-1/2}\int_{L_\lambda}[a_1C_1(\lambda,\mu)+a_2C_2(\lambda,\mu)]e^{-i\lambda x}d\lambda = 0\quad (x>0)\\
&(2\pi)^{-1/2}\int_{L_\lambda}\frac{i\gamma}{\lambda}[b_1C_1(\lambda,\mu)+b_2C_2(\lambda,\mu)]e^{-i\lambda x}d\lambda = 0\quad (x>0)
\end{aligned} \tag{1.7}$$

where

$$a_m = \frac{1}{C_{44}^{(1)}(1+q_{1m})} + \frac{\zeta_{1m}-\zeta_{2m}}{C_{44}^{(2)}(\omega_{21}-\omega_{22})}$$
$$b_m = \frac{\omega_{1m}q_{1m}}{C_{44}^{(1)}(1+q_{1m})} + \frac{\omega_{22}q_{22}\zeta_{2m} - \omega_{21}q_{21}\zeta_{1m}}{C_{44}^{(2)}(\omega_{21}-\omega_{22})}$$
$$\zeta_{1m} = \frac{\omega_{22}+\omega_{1m}}{1+q_{21}},\quad \zeta_{2m} = \frac{\omega_{21}+\omega_{1m}}{1+q_{22}}$$
$$C_m(\lambda,\mu) = C_{44}^{(1)}(1+q_{1m})A_{1m}(\lambda,\mu),\qquad m=1,2$$

$$p^*(x,\mu) = (2\pi)^{-1/2}\int_{-\infty}^{\infty}p(x,y)e^{i\mu y}dy$$

We will denote the Fourier transforms with respect to the x and y coordinates of the shear stresses τ_{xz} and normal stresses σ_z on the continuation of the crack by $T_+(\lambda,\mu)$ and $S_+(\lambda,\mu)$ and the transforms of the displacements of the crack edges $u_1(x,y,0) - u_2(x,y,0)$ and $w_1(x,y,0) - w_2(x,y,0)$ by $U_-(\lambda,\mu)$ and $W_-(\lambda,\mu)$ respectively. The system of coupled integral equations (1.7) then reduces to a Riemann matrix problem with a complex variable λ and a real parameter μ

$$D\gamma G(\lambda,\mu)F_-(\lambda,\mu) = F_+(\lambda,\mu) - Q(\lambda,\mu),\quad \lambda\in L_\lambda \tag{1.8}$$

$$G(\lambda,\mu)=\begin{Vmatrix} g_1 & i\lambda g_3/\gamma \\ -i\gamma g_3/\lambda & g_2 \end{Vmatrix},\quad Q(\lambda,\mu)=\begin{Vmatrix} 0 \\ P_-(\lambda,\mu) \end{Vmatrix}$$

$$F_-(\lambda,\mu)=\begin{Vmatrix} U_-(\lambda,\mu) \\ W_-(\lambda,\mu) \end{Vmatrix},\quad F_+(\lambda,\mu)=\begin{Vmatrix} T_+(\lambda,\mu) \\ S_+(\lambda,\mu) \end{Vmatrix}$$

$$P_-(\lambda,\mu)=(2\pi)^{-1/2}\int_{-\infty}^{0} p^*(x,\mu)e^{i\lambda x}dx$$

The constant D and the quantities g_k $(k = 1, 2, 3)$ which occur in the matrix coefficient of problem (1.8) are determined solely by the moduli of elasticity of the materials and have the form (the constant D, which is not involved in the subsequent calculations, is omitted)

$$\begin{Vmatrix} g_1 \\ g_2 \\ g_3 \end{Vmatrix}=(\xi_2^2-\eta_2^2)\begin{Vmatrix} \xi_1\Omega_1 \\ C_{33}^{(1)}\Omega_1 \\ \xi_1-\eta_1 \end{Vmatrix}+(\xi_1^2-\eta_1^2)\begin{Vmatrix} \xi_2\Omega_2 \\ C_{33}^{(2)}\Omega_2 \\ \eta_2-\xi_2 \end{Vmatrix} \tag{1.9}$$

$$\xi_j=(C_{11}^{(j)}C_{33}^{(j)})^{1/2},\quad \eta_j=C_{13}^{(j)},\quad \Omega_j=\omega_{j1}+\omega_{j2},\quad j=1,2$$

Only the sums of the characteristic numbers ω_{jn} $(j, n = 1, 2)$ occur in expressions (1.9), and therefore, in spite of the fact that they may be complex, the values of g_k are real for any pair of materials. Moreover, it follows from inequalities (1.6) that g_1 and g_2 are positive. We also note that, if the materials of the half-spaces are identical, then $g_3 = 0$, which follows from (1.9). In this case, $G(\lambda, \mu)$ has a diagonal form and, consequently, matrix problem (1.8) decomposes into two scalar problems.

2. The factorization of its coefficient matrix is a key step in the solution of Eq. (1.8). For this purpose, we multiply (1.8) by the constant matrix $S = \operatorname{diag}(g_1^{-1}, g_2^{-2})$ and, as a result, we obtain

$$DR(\lambda,\mu)F_-(\lambda,\mu)=SF_+(\lambda,\mu)-SQ(\lambda,\mu),\quad \lambda\in L_\lambda \tag{2.1}$$

where

$$R(\lambda,\mu)=\gamma Z(\lambda,\mu)$$

$$Z(\lambda,\mu)=SG(\lambda,\mu)=\begin{Vmatrix} 1 & i\lambda g_3(\gamma g_1)^{-1} \\ -i\gamma g_3(\lambda g_2)^{-1} & 1 \end{Vmatrix} \tag{2.2}$$

We note that the matrix $Z(\lambda, \mu)$ has constant eigenvalues

$$\Lambda_{1,2}=1\mp\beta,\quad \beta=g_3(g_1g_2)^{-1/2} \tag{2.3}$$

(where $|\beta| < 1$) by virtue of inequalities (1.6)) and has the polynomial commutant

$$B(\lambda,\mu)=\frac{1}{\lambda\gamma}\begin{Vmatrix} 0 & i\lambda^2/g \\ -i(\lambda^2+\mu^2)g & 0 \end{Vmatrix},\quad g=(g_1/g_2)^{1/2}$$

The matrix (2.2) is then factorized using the formulae [5]

$$Z(\lambda,\mu)=X_+(\lambda,\mu)X_-^{-1}(\lambda,\mu) \tag{2.4}$$

$$X^{\pm 1}(\lambda,\mu)=\varphi^{\pm 1}(\lambda,\mu)\{I\operatorname{ch}[\gamma\psi(\lambda,\mu)]\pm B(\lambda,\mu)\operatorname{sh}[\gamma\psi(\lambda,\mu)]\}$$

Here I is the identity matrix and the functions $\varphi(\lambda, \mu)$ and $\psi(\lambda, \mu)$ satisfy the two scalar equations

$$\varphi_+(\lambda,\mu)\varphi_-^{-1}(\lambda,\mu)=\Delta^{1/2},\quad \lambda\in L_\lambda \tag{2.5}$$

$$\psi_+(\lambda,\mu)-\psi_-(\lambda,\mu)=\varkappa/\gamma,\quad \lambda\in L_\lambda \tag{2.6}$$

the right-hand sides of which are determined by eigenvalues (2.3)

$$\Delta = \Lambda_1\Lambda_2 = 1-\beta^2, \quad \varkappa = \tfrac{1}{2}\ln(\Lambda_1/\Lambda_2) = \tfrac{1}{2}\ln\frac{1-\beta}{1+\beta}$$

The solutions of problems (2.5) and (2.6) are found using a well-known method [6] and expressed by the formulae

$$\varphi_+(\lambda,\mu) = 1, \quad \varphi_-(\lambda,\mu) = (1-\beta^2)^{-1/2} \tag{2.7}$$

$$\psi_\pm(\lambda,\mu) = i\varepsilon\gamma^{-1}\ln[(\lambda+\gamma)/(\pm i|\mu|)]$$

The bielastic constant $\varepsilon = (2\pi)^{-1}\ln[(1-\beta)/(1+\beta)]$ which appears here has the same structure as in plane problems concerning interfacial cracks in isotropic media [7]. However, the constant β, which is defined by formula (2.3), takes the part of the Dundurs constant [8] in the case of transversely isotropic media.

The factorization of the coefficient matrix of problem (2.1) therefore has the form

$$R(\lambda,\mu) = R_+(\lambda,\mu)R_-(\lambda,\mu), \quad R_\pm(\lambda,\mu) = (\lambda \pm i|\mu|)^{1/2}X_\pm^{\pm 1}(\lambda,\mu) \tag{2.8}$$

Equation (2.1) can then be represented in the form

$$DR_-(\lambda,\mu)F_-(\lambda,\mu) + Q_-(\lambda,\mu) = R_+^{-1}(\lambda,\mu)SF_+(\lambda,\mu) - Q_+(\lambda,\mu) \tag{2.9}$$

where

$$Q_\pm(\lambda,\mu) = \pm\frac{1}{2\pi i}\int_{L_\alpha} R_+^{-1}(\alpha,\mu)SQ(\alpha,\mu)\frac{d\alpha}{\alpha-\lambda} \tag{2.10}$$

The contour L_α is situated between the real axis and the contour L_λ.

Using the principle of analytic continuation, Liouville's theorem and the condition that the stresses diminish at infinity, we obtain the relation

$$F_+(\lambda,\mu) = S^{-1}R_+(\lambda,\mu)Q_+(\lambda,\mu)$$

Hence it follows that the stresses in the continuation of the crack have the form

$$\left\|\begin{matrix}\tau_{xz}(x,y,0)\\ \sigma_z(x,y,0)\end{matrix}\right\| = \frac{1}{\pi}\int_0^\infty\left[\int_{L_\lambda} S^{-1}R_+(\lambda,\mu)Q_+(\lambda,\mu)e^{-i\lambda x}d\lambda\right]\cos\mu y\, d\mu \tag{2.11}$$

3. We shall now construct the asymptotic forms of the stresses (2.11) when $x \to +0$. According to a theorem of the Abel type [6], they are determined by the asymptotic forms of the integrands when $\lambda \to \infty$.

Using formulae (2.4), (2.7) and (2.8) as $\lambda \to \infty$, we find

$$\frac{R_+(\lambda,\mu)}{\alpha-\lambda} \sim -\frac{1}{2}\left[\left(\frac{2\lambda}{i\mu}\right)^{i\varepsilon}\Psi^+ + \left(\frac{2\lambda}{i\mu}\right)^{-i\varepsilon}\Psi^-\right]\lambda^{-1/2}, \quad \Psi^\pm = \left\|\begin{matrix}1 & \pm ig^{-1}\\ \mp ig & 1\end{matrix}\right\|$$

Then, by taking account of relation (2.10), applying the theorems on residues and using the value of the integral [9]

$$\int_0^\infty t^{\nu-1}e^{-tx}dt = x^{-\nu}\Gamma(\nu) \quad (x>0)$$

from (2.11), we arrive at the following representation of the stresses close to the crack tip

$$\left\|\begin{matrix}\tau_{xz}(x,y,0)\\ \sigma_z(x,y,0)\end{matrix}\right\| \sim \frac{1}{2\pi^2}(\Phi^+ + \Phi^-), \quad \Phi^\pm = S^{-1}\Psi^\pm\Gamma\left(\frac{1}{2}\pm i\varepsilon\right)\int_0^\infty f(\mu)\left(\frac{\mu}{2}\right)^{\mp i\varepsilon}\cos\mu y\, d\mu\, x^{-1/2\mp i\varepsilon} \tag{3.1}$$

$$S^{-1} = \operatorname{diag}(g_1,g_2), \quad f(\mu) = \left\|\begin{matrix}(g_1g_2)^{-1/2} & f_2(\mu)\\ g_2^{-1} & f_1(\mu)\end{matrix}\right\| = i^{1/2}\operatorname{ch}\pi\varepsilon\int_{L_\lambda} R_+^{-1}(\alpha,\mu)SQ(\alpha,\mu)d\alpha$$

Using formulae (2.4), (2.7) and (2.8), the elements of the column matrix $f(\mu)$ can be expressed in terms of the Fourier transform $P_-(\alpha, \mu)$ of the normal load

$$f_1(\mu) = i^{1/2} \operatorname{ch} \pi\varepsilon \int_{L_\alpha} \frac{P_-(\alpha,\mu)}{(\alpha+i\mu)^{1/2}} \cos\left(\varepsilon \ln \frac{\alpha+\gamma}{i\mu}\right) d\alpha$$

$$f_2(\mu) = i^{1/2} \operatorname{ch} \pi\varepsilon \int_{L_\alpha} \frac{\alpha P_-(\alpha,\mu)}{\gamma(\alpha+i\mu)^{1/2}} \sin\left(\varepsilon \ln \frac{\alpha+\gamma}{i\mu}\right) d\alpha$$

We now introduce the functions

$$r_n(y) = \Gamma\left(\frac{1}{2}+i\varepsilon\right) \int_0^\infty f_n(\mu) \left(\frac{\mu}{2}\right)^{-i\varepsilon} \cos\mu y \, d\mu \quad (n=1,2) \tag{3.2}$$

The asymptotic forms of the stresses when $x \to +0$ then take the form

$$\begin{aligned} \sigma_z(x,y,0) &\sim [K_1(y)\cos(\varepsilon \ln x) - K_2(y)\sin(\varepsilon \ln x)]x^{-1/2} \\ \tau_{xz}(x,y,0) &\sim g[K_1(y)\sin(\varepsilon \ln x) + K_2(y)\cos(\varepsilon \ln x)]x^{-1/2} \end{aligned} \tag{3.3}$$

The stress intensity factors $K_1(y)$ and $K_2(y)$ are calculated from the formulae

$$K_1(y) = \frac{1}{\pi^2}[\operatorname{Re} r_1(y) + \operatorname{Im} r_2(y)], \quad K_2(y) = \frac{1}{\pi^2}[\operatorname{Re} r_2(y) - \operatorname{Im} r_1(y)] \tag{3.4}$$

The three-dimensional stress fields in the neighbourhood of the crack tip located at the interface of two different transversely isotropic materials are therefore oscillating. The asymptotic formulae (3.3) have the same structure as in the case of the plane [7] and axially symmetric [10] problems of interfacial cracks in isotropic media.

4. As an example, we will now consider the case when point normal forces of magnitude P are applied to the crack edges on the x axis at a distance a from its tip, that is, $p(x, y) = P\delta(x + a)\delta(y)$ and, consequently, $P_-(\lambda, \mu) = Pe^{-i\lambda a}/(2\pi)$. The function (3.2) can then be represented in the form of single quadratures [1]

$$\begin{aligned} r_1(y) &= Ph(\varepsilon) \int_1^\infty \frac{\cos[\varepsilon l(\xi)]}{\sqrt{\xi-1}} \chi(\xi,y) d\xi \\ r_2(y) &= Ph(\varepsilon) \left\{ \int_1^\infty \frac{\xi \sin[\varepsilon l(\xi)]}{(\xi-1)\sqrt{\xi+1}} \chi(\xi,y) d\xi + 2^{-1/2} \chi(1,y) \operatorname{th} \pi\varepsilon \right\} \end{aligned} \tag{4.1}$$

$$h(\varepsilon) = 2^{i\varepsilon}(1/2 - i\varepsilon) \operatorname{ch} \pi\varepsilon, \quad l(\xi) = \ln(\xi + \sqrt{\xi^2-1}), \quad \chi(\xi,y) = (y^2 + a^2\xi^2)^{-3/4 + i\varepsilon/2} \cos\left[\left(\frac{3}{2} - i\varepsilon\right) \operatorname{arctg} \frac{y}{a\xi}\right]$$

Table 1

Materials	a = 0,25		0,5		1	
	K_1^*	K_2^*	K_1^*	K_2^*	K_1^*	K_2^*
SiO_2–Mg	7,909	1,437	2,806	0,4533	0,9949	0,1409
$BaTiO_3$–Mg	7,805	2,499	2,788	0,7896	0,9945	0,2457
$BaTiO_3$–Zn	7,925	1,192	2,808	0,3759	0,9949	0,1168
Zn–Mg	7,912	1,389	2,806	0,4381	0,9949	0,1362
Co–Cd	7,844	2,168	2,794	0,6848	0,9947	0,2130
Co–Zn	7,827	2,320	2,792	0,7329	0,9946	0,2280

The results of the calculations of the stress intensity factors using formulae (3.4) and (4.1) for several pairs of transversely isotropic materials and values of the parameter $a = 0.25$, 0.5 and 1.0 are presented in Table 1, where $K_1^* = \pi^2 K_1(0)/P$, $K_2^* = \pi^2 K_2(0)/P$.

The values of the elasticity constants from [11] were used in the calculations. The data presented in Table 1 enable us to conclude that, for all the pairs of materials which have been considered, as well as in the case of isotropic media [1], the magnitudes of K_1^* hardly differ from the stress intensity factor $\pi^2 K_1(0)/P = a^{-3/2}$ in the problem of a homogeneous isotropic space with a semi-infinite crack [12]. The coefficients K_2^* take different values depending on the combination of materials, and increase when the distance from the point of application of the load to the fracture front decreases.

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